Warm-Up

- Begin reading the set of six activities that appear on the green pages of your handout packet.

- As you read, consider the following question: What learning opportunities are afforded by this set of activities?
Content-focused Methods Courses: Integrating Pedagogy and Mathematical Content

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Overview of Session

- Describe and provide a rationale for content-focused methods courses

- Consider the potential for content-focused methods courses to develop teachers' knowledge base for teaching by:
  - analyzing a set of activities used in an iteration of a course that we designed, taught, and studied
  - analyzing data collected from an iteration of the same course

- Reflect on the features of content-focused methods courses
Mathematical Knowledge for Teaching: The Invisible Secondary Gap

Subject Matter Knowledge

Common Content Knowledge (CCK)

Horizon Knowledge

Specialized Content Knowledge (SCK)

Pedagogical Content Knowledge

Knowledge of Content and Teaching (KCT)

Knowledge of Content and Students (KCS)

Knowledge of Content and Curriculum

Many courses in mathematics

Course(s) in math teaching methods

But no systematic opportunities here
One Possible Solution: A Content-Focused Methods Course

1. Focuses on a narrow slice of mathematical content or process central to developing mathematical proficiency in grades 7-12 (rather than sampling across diverse mathematical topics)

2. Uses a guiding inquiry to frame & motivate the course and provides a unifying thread

3. Attends to both content and pedagogy and the ways in which pedagogy can support the learning of content

4. Brings together teachers with varying levels of experience and prior knowledge
Four Key Features of a Content-Focused Methods Course

1. Focuses on mathematical content central to developing mathematical proficiency in grades 7-12 (e.g., functions; reasoning-and-proving)
   - Cuts across grade levels
   - Presents a challenge for students (and teachers)
   - Is complex enough so as to sustain weeks (or months) of teacher education or professional development
   - Is identified in standards documents
Four Key Features of a Content-Focused Methods Course

2. Uses a guiding inquiry to frame & motivate the course and provides a unifying thread
   - Provides coherence and focuses investigation throughout the course
   - Exposes teachers’ initial conceptions about the content focus, including misconceptions
   - Provides the opportunity to refine and elaborate initial understandings
Four Key Features of a Content-Focused Methods Course

3. Attends to both content and pedagogy and the ways in which pedagogy can support the learning of content

- Experiencing content as learners and as teachers helps to make sense of student thinking and related teacher moves
- Models a student-centered pedagogy with the content of secondary mathematics, providing teachers with a usable model for their own classrooms
Four Key Features of a Content-Focused Methods Course

4. Brings together teachers with varying levels of experience and prior knowledge
   - Likely to lead to greater diversity in approaches to mathematical tasks
   - Can bring different viewpoints on pedagogy and issues of student learning
   - Can also illuminate similarities across diverse populations and contexts (e.g., a wide range of students holding a particular misconception)
Proportional Reasoning
- Are all fractions ratios, and are all ratios fractions?
- What is proportional reasoning?

Algebra as the Study of Patterns and Functions
- What is a function?

Geometry and Measurement
- What are the opportunities to prove in this geometry task?
- What is a proof?

Reasoning-and-Proving
- What is reasoning-and-proving?
- How do secondary students benefit from engaging in reasoning-and-proving?
- How can teachers support the development of students’ capacity to reason-and-prove?
Considering How the Key Features Play Out

Examining an Activity Sequence from the Algebra Course
Four Key Features of a Content-Focused Methods Course

1. **Content focus**: Algebra as the study of patterns and functions

2. **Guiding inquiry**: What is a function? (specifically, how do we define function and what are examples and non-examples?)

3. **Integrating content & pedagogy**: Solved mathematical tasks, analyzed narrative and video cases, examined student work, and read key research on learning about function

4. **Diverse population**: About 50% preservice, 50% practicing
   About 75% secondary, 25% elementary or special education
Considering Opportunities to Learn

The green pages of your handout packet contain six activities that make up a “constellation” in the functions course. Continue reading this set of activities, and consider the following question:

What learning opportunities are afforded by this set of activities?
Opportunities to Learn

- **Mathematics**
  - looking for and generalizing the underlying structure of a visual pattern (Activity 2)
  - connecting representations (Activity 2)
  - defining function (Activity 5)
  - considering the role of definition in mathematics (Activity 5)

- **Pedagogy**
  - change to- distinguishing tasks based on their thinking demand (Activity 1)
  - connecting tasks with opportunities to learn (Activities 3, 4)
  - identify factors that impact the implementation of high level tasks (Activity 3)
  - asking questions that assess and advance student thinking (Activity 3, 6)
Course Goals

Mathematical

- Distinguish between functions & non-functions; linear & nonlinear functions; and proportional & nonproportional functions
- Solve pattern & function problems with linear and nonlinear relationships, using recursive or closed form terminology and notation
- Recognize equivalence of different representations of functions, use them to answer questions and solve problems, & make connections between representations
- Interpret & construct qualitative graphs

Pedagogical

- Support the development of students’ understanding of functions by encouraging and facilitating rich mathematical discussions by:
  - Pressing for clarification and explanation
  - Requiring justifications of proposals and challenges
  - Recognizing and challenging misconceptions
  - Demanding evidence for claims and arguments
  - Interpreting and revoicing students’ statements
  - asking questions that probe and extend student thinking
- Select and enact cognitively challenging mathematical tasks
- Identify factors that impact the maintenance and decline of cognitive demands during implementation
Work on the guiding inquiry
Studying Teacher Learning: Considering Two Pre/Post Items

- Written assessment, interview items, teacher work analysis, and discourse analysis were used to assess learning.
  - Both mathematical and pedagogical goals were assessed.

- Examine the two tasks shown on the yellow page of your handout packet, and consider the following question:
  - What is the range of performance that these tasks are likely to elicit?
## Studying Teacher Learning: Considering Two Pre/Post Items

### Pentagon Pattern Task

<table>
<thead>
<tr>
<th>Rubric Score</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>incorrect</td>
</tr>
<tr>
<td>1</td>
<td>no connection between the solution and the visual pattern</td>
</tr>
<tr>
<td>2</td>
<td>weak or flawed explanation with some connection to the visual pattern</td>
</tr>
<tr>
<td>3</td>
<td>sound generalization with a partial connection to the visual pattern</td>
</tr>
<tr>
<td>4</td>
<td>generalization whose representations were well-connected to the visual pattern</td>
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</table>

### Defining Function Task

<table>
<thead>
<tr>
<th>Rubric Score</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>Incorrect</td>
<td>does not include the idea of univalence, or makes erroneous statements (e.g., functions must be linear relationships)</td>
</tr>
<tr>
<td>Inconclusive</td>
<td>not enough information present to suggest that the teacher included the univalence criterion.</td>
</tr>
<tr>
<td>Correct</td>
<td>includes the idea of univalence and does not explicitly rule out arbitrariness</td>
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</table>
Studying Teacher Learning: Considering Two Pre/Post Items

### Pentagon Pattern Task

<table>
<thead>
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### Defining Function Task

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<tbody>
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<tr>
<td>Correct</td>
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<td>20</td>
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Studying Teacher Learning: Pivotal Course Activities

- Teachers cited a wide range of course activities as contributing to their learning

- All discussed learning about mathematics, the teaching of mathematics, and students as learners of mathematics
Two Cases of Teacher Learning

- Olivia: Practicing Elementary Teacher
- Carl: Preservice Secondary Teacher
Tracing Olivia's Learning

**Pre-Course Assessments**

A function is a relationship that can exist for a variety of numbers. In other words, different numbers can be used in the place of a variable, and the relationship can be maintained.

**Includes univalence?** ✗

**Doesn't rule out arbitrariness?** ✗

Example of function:

\[ y = 2x \]

\[ y \text{ is different, depending on the value of } x \]

Example of non-function:

\[ x + 4 = 20 \]

This is not a function because \( x \) can only have one value → 16

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**Course activities identified as contributing to learning:**

- **Hair Growth Task** (Class 5)
- **Cal's Dinner Card Deals** (Class 7)
- **A Function is a Mail Carrier** (Class 9)
- Creating a definition of function (several classes)

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**Post-Course Assessments**

A function occurs when 2 variables vary together. One variable is dependent on the other variable. For each value of the independent variable, there must be only one value of the dependent variable.

**Includes univalence?** ✓

**Doesn't rule out arbitrariness?** ✓

Example of function:

\[ f(x) = 4x + 2 \]

The initiation fee for getting into a club is $2. At each meeting, the dues are $4. How much money will be spent after any given meeting?

Example of non-function:

\[ y^2 = x \]

- ✗: Absent
- ✓: Present
- ?: Inconclusive
Olivia: Practicing elementary teacher (post-course interview)

“One thing I also liked about the class is that we really worked on developing our understanding of math AND connecting it to teaching, like through the case studies. And, there aren’t very many classes that do that, that I’ve taken.

I mean, I feel like I need to really take classes like where I am developing my understanding of the math. And it seems like at this point, they expect you to know it, and it’s more these general classes about teaching of the math, or general topics- and I think the two go hand in hand- really learning about the math and understanding it, then looking at well, how is that taught in the classroom?

And [the instructor] really did that a lot, you know, we looked at the tasks first. So, we understood basically, what this task was about, the math that was involved, and then how a teacher was presenting the task, and how students in the task interpreted it, and maybe compare in your mind, “Well, you know, that’s how I thought of it.” So that’s really important. Or, it’s effective I think for teachers because both of them are really important and connecting them are important.”
Tracing Carl's Learning

Pre-Course Assessments

A function is a sentence/expression/relationship between an input variable and an output variable.

Includes univalence?  **×**

Doesn't rule out arbitrariness?  **✓**

Example of function:

\[ f(x) = x^2 \]

Example of non-function:

A mathematical sentence where the input/independent variable has no bearing/effect on the output/dependent variable; i.e., "# of buttons on my calculator vs. SAT Score"

Course activities identified as contributing to learning:

A Function is a Mail Carrier (Class 9)

Discuss 3 textbook definitions (Class 9)

Post-Course Assessments

A function is a correspondence between two sets of values A & B where each value of A corresponds exactly with one value of B.

Includes univalence?  **✓**

Doesn't rule out arbitrariness?  **✓**

Example of function:

\[ y = 2x + 8 \]

Example of non-function:

\[ x^2 + y^2 = 1 \]

**×**: Absent

**✓**: Present

?**: Inconclusive
Carl: Preservice secondary teacher (post-course interview)

“I have a clearer definition of what a function is…I think most of us came in to the class having worked with functions before obviously and, doing vertical line tests to see if something in the function maybe is not a function.

But I don’t think a lot of us had a really SOLID DEFINITION in our heads of what a function is. And I think that the class kinda helped us revise our own inkling of what a function is. The thing about functions is that correspondence between two different sets of quantities.

Before the course, if someone had asked me, “What is a function?,” I couldn’t say…I would’ve said something about the vertical line test. I would’ve said something about an equation, [or] function notation. But I don’t think I could’ve really given a really direct answer.

After the course, I think I can.”
Take a few minutes to consider...

- What do you see as the advantages and disadvantages of the content-focused methods course model?
- How might you integrate aspects of the model into the methods courses that you teach?

1. Focuses on a narrow slice of mathematical content or process central to developing mathematical proficiency in grades 7-12 (rather than sampling across diverse mathematical topics)

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